Asymmetric behavior in consecutive phase space point spacings and nonintegrability

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Hamilton 系のカオスを判定し，考察するのに時系列に並んだ位相プロット間隔の伸び率と縮み率の対称性の破れに着目する．対称性の破れ具合に対して相関係数を導入する事で，可積分性から非可積分性への転移過程を定量的に評価する．また，Lyapunov 指数との比較を行う事でその特徴を示す．

1 Consecutive phase space point spacings

First, a spacing of consecutive phase space points on the Poincaré section plane is defined by $L_n \equiv \sqrt{(q_{n+1}-q_n)^2+(p_{n+1}-p_n)^2}$. Here, n enumerates a set of points on the Poincaré section plane. Following $SU(3)$ Hamiltonian was used in analyzing L_n.

$$H(q^{(1)},p^{(1)};q^{(2)},p^{(2)}) = 2\epsilon^{(0)}r^2 + \epsilon^{(1)}\frac{(q^{(1)^2}+p^{(1)^2})}{2} + \epsilon^{(2)}\frac{(q^{(2)^2}+p^{(2)^2})}{2}$$
$$+V^{(1)}\frac{2(\mathcal{N}-1)}{\mathcal{N}}r^2(q^{(1)^2}-p^{(1)^2}) + V^{(2)}\frac{2(\mathcal{N}-1)}{\mathcal{N}}r^2(q^{(2)^2}-p^{(2)^2}), \tag{1}$$

where $r \equiv \frac{1}{2}\sqrt{2\mathcal{N}-q^{(1)^2}-p^{(1)^2}-q^{(2)^2}-p^{(2)^2}}$, $V^{(1)}$ and $V^{(2)}$ mean strength of perturbation interaction in each degree of freedom. When analyzing it thereafter, $V^{(1)}=V^{(2)}=V$. In our numerical calculation, we used $\mathcal{N}=30$ and the single particle energies were $\epsilon_0=0$, $\epsilon_1=1$ and $\epsilon_2=2$, with which our energy scale is non-dimensional.

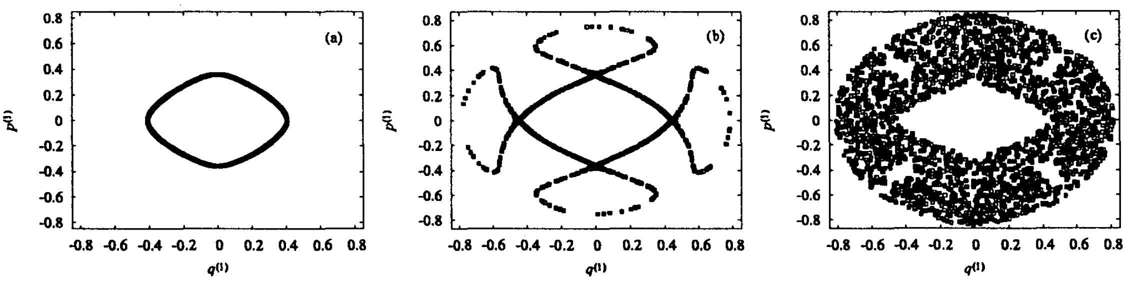

Figure 1: Poincaré section plane for Eq. (1). (a) $V=-0.02$, (b) $V=-0.042$, (c) $V=-0.093$.

2 Symmetry violation in consecutive phase space point spacings

In this section, we demonstrate the characterization of chaos by exploiting a correlation coefficient for a rate of elongations and contractions of L_n. [1] Rates of elongations and contractions are given as $R_n \equiv \frac{L_{n+1}}{L_n} > 1$, $r_n \equiv \frac{L_{n+1}}{L_n} < 1$. An average rates of elongations for a $2N$ number of phase space point spacings is given by $\bar{R} \equiv \frac{1}{N}\sum_{n=1}^{N} R_n$. One may introduce a correlation coefficient for R_n and r_n given as

$$C_{R_n r_n} = \frac{(R_1-\bar{R})(r_1-\bar{r})+\cdots+(R_N-\bar{R})(r_N-\bar{r})}{\sqrt{(R_1-\bar{R})^2+\cdots+(R_N-\bar{R})^2}\sqrt{(r_1-\bar{r})^2+\cdots+(r_N-\bar{r})^2}}. \tag{2}$$

Now, let us begin our analysis by considering the regular-to-chaos transition as shown in Figs. 1(a)-(c). Since the periodic sequence for consecutive phase space point spacings is realized in a

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case with weak interaction, R_n and r_n are symmetric with respect to $\frac{L_{n+1}}{L_n} = 1$ as seen from Fig. 2(a). In this case, the correlation coefficient is close to -1 as shown in the data of $V = -0.02$ of Fig. 3(a). In Fig. 2, it should be noted that the time series of R_n and r_n have described as independent of each other. In a case of the hyperbolic orbit as shown in Fig. 1(b), the behavior of R_n and r_n is illustrated in Fig. 2(b). In Figs. 2(b) and 3(a), the identification of an inside orbit and an outside orbit of hyperbolic point becomes possible in the vicinity where $n \approx 1000$ was passed. When the interaction increases and more repulsion effects appearing between consecutive phase space points, one may not expect symmetric behaviors between R_n and r_n, which is shown in Fig. 2(c). In this case, as shown in the data of $V = -0.093$ of Fig. 3(a), the correlation coefficients are close to 0. Incidentally, $C_{R_n r_n} = -1$ and $C_{R_n r_n} = 0$ are correspondent to negative and positive values of the Lyapunov exponent, as represented in Figs. 3(a)-(c). [1]

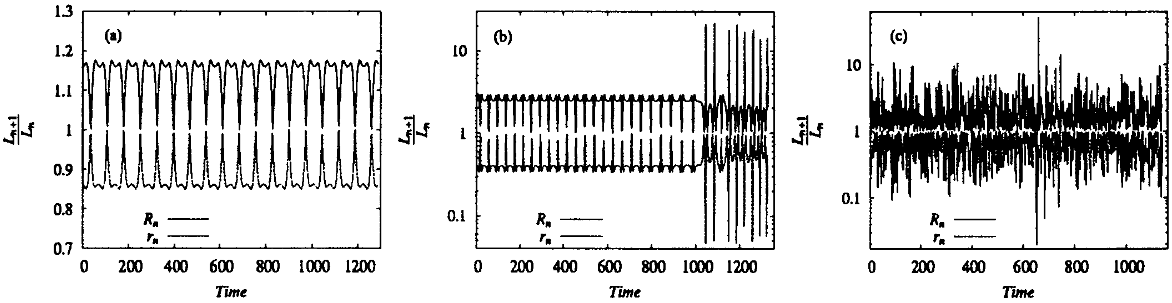

Figure 2: Rates of the elongations and contractions of L_n in the cases with (a) $V = -0.02$, (b) $V = -0.042$, and (c) $V = -0.093$.

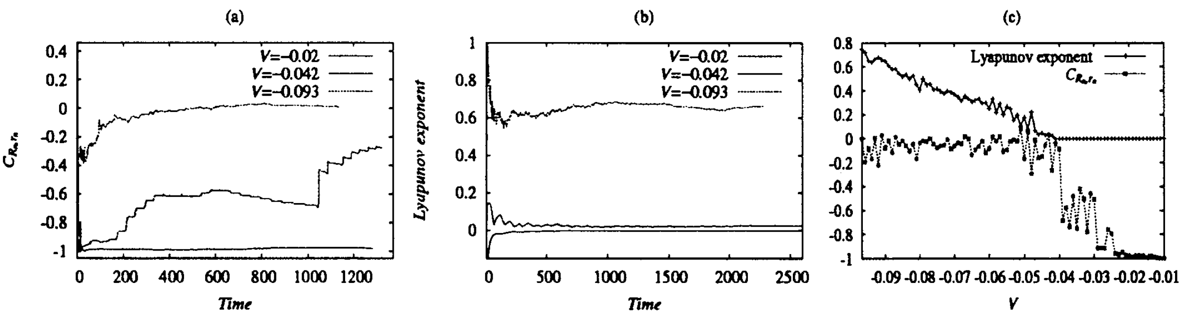

Figure 3: (a) Correlation coefficients for Figs. 1(a)-(c) and (b) Lyapunov exponents for Figs. 1(a)-(c). (c) Correlation coefficients and Lyapunov exponents at the each value of V.

3 Conclusion

We conclude that the result of this analysis provides us with a new method to consider how the nonintegrabilities depend on the degree of symmetry violations between R_n and r_n. Moreover, the thing that a negligible difference of unstable orbits that was not able to be identified by the Lyapunov exponent was able to be identified by using the correlation coefficient was confirmed. [1] It is enumerated to derive the distribution function that can statistically describe the symmetry violation as a problem in the future.

References

[1] S. Fujiwara, to be published in *The bulletin of Hiroshima Mercantile Marine College* (2005).